Is a uniform price of Carbon desirable?

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The case for a uniform price of Carbon

- Climate policy goals should be reached with a uniform price on CO2 emissions. With a "correct" price, emission reductions are realized with minimal costs to society. Sector-specific rules are superfluous or even harmful.
- A uniform price provides "correct" incentives for firms confronted with switching to greener technologies or paying for emissions. No need for the government to provide further incentives for the use of green technologies.

Political reality: Sector-specific rules all over the place

- European Union: emission trading system covering e.g. electricity and heat generation, aluminium, cement, steel works... Separate emission trading system (EU ETS II) planned covering buildings and road transport, CO2 emission performance standards for cars and vans.
- Germany: Green taxes covering fossil fuels and electricity, regulation of heating systems.

Questions

- Is a sector-specific approach to climate policy an indication of a "political failure"?
- Is there a plausible justification for a sector-specific approach?

To obtain answers study the desirability of a uniform price of Carbon using tools from the theory of taxation.

This paper

- Use a model so that
  - Individuals differ in their incomes and in their preferences for green versus brown consumption goods.
  - The incentive of firms to reduce emissions depend on the CO2 prices they are facing.
- This framework nests as special cases
  - The Mirrleesian model of optimal income taxation
  - Ramsey's model of optimal sector-specific taxation
  - The partial equilibrium model, which is often used to justify a uniform price of Carbon

Main results

Is there a plausible justification for sector-specific Carbon prices/ taxes?

1. There are somewhat restrictive conditions under which a uniform price is justified in the sense that any departure from it implies a violation of Pareto-efficiency.
2. More generally a uniform price can be justified under an assumption of *distributive indifference* and with *proportional fiscal externalities.*
3. In the presence of a non-linear income tax, sector specific polices can be justifiable even under *distributive indifference*.
4. With distributive concerns, sector specific rules and hence a departure from a uniform price can be justified.

Related literatures

- Regulation of externalties: Weitzman (1974), ... partial equilibrium model
- Sector-specific taxation: Ramsey (1927), Diamond and Mirrless (1971), Diamond (1975),...
- General equilibrium tax incidence: Harberger (1962), Bovenberg and Goulder (1994), Tsyvinki et al. (2020) ...
- Direct versus indirect taxation: Atkinson-Stiglitz (1976), Saez (2002), Hellwig and Werquin (2023)
- Optimal externalities and optimal taxation: Cremer, Gahvari and Ladoux (1998), Golosov et al. (2014), Jacobs and de Mooij (2015), Jacobs and van der Ploeg (2019), Pai and Strack (2022) and Ahlvik et al. (2024).
- Distributive consequences of carbon pricing: Känzig (2023)

Households I

- Unit mass of households with preferences $u(x_c, \chi(\beta x_g, x_b)) - k(y, \omega)$.
- Consumption utility u assumed to be homothetic

  x_c: unspecific consumption good

  χ: subutility from combining a green (x_g) and a brown (x_b) good.

  β: strength of the preference for the green vs the brown good.

- Effort cost function k

  y: labour supply in efficiency units,

  ω: productive ability, affects the marginal effort costs, $k_{12} < 0$.

- Budget constraint:

$$q_c x_c + q_g x_g + q_b x_b \quad \leq \quad p_w\, y_l - T_l(p_w\, y_l) + s\, \Pi^E + \mathcal{R}^E \ , \tag{1}$$

where $q_j = (1 + t_j)p_j$, for $j \in \{c, g, b\}$.

Households II

Implications:

- Engel curves are linear. Heterogeneity in the composition of the consumption basket only due to heterogeneity in the preference for green versus brown consumption goods, parameterized by β.
- A hypothetical redistribution of one unit of income from a high β to a low β person reduces the demand for emission intensive goods.

For some of the results impose **Assumption 1:**

- $u(x_c, \chi(\beta x_g, x_b)) = x_c^{1-\nu} \chi(\beta x_g, x_b)^{\nu}$
- $\chi = \left(\beta x_g^{1-\varepsilon_\chi} + x_b^{1-\varepsilon_\chi}\right)^{\frac{1}{1-\varepsilon_\chi}}$.
- ν small.

Think of the green and the brown sector as targets for specific policies vis à vis *the rest of the economy*.

Firms I

The profit-maximization problem of a generic firm in sector $j \in \{c, b, g\}$ is to choose labour demand l and $R\&D$ effort r to maximize

$$p_j \; \alpha \; f_j(l) - p_w \; l - t_{je}\Big(e_{j0} - a_j(r)\Big) \; \alpha f_j(l) - p_c \; \gamma \; r \; .$$

- The abatement function $a_j : r \mapsto a_j(r)$, with $a_j(0) = 0$, gives the decrease in the emission intensity of production.
- The production function f_j is assumed to satisfy the usual Inada conditions.
- Firms differ in factor productivity α and abatement costs γ.
- There are, possibly sector-specific, taxes/ prices for emission permits t_{je}.

Firms II

Lemma

Consider a firm in sector $j \in \{c, b, g\}$ with characteristics $\theta_j = (\alpha_j, \gamma_j)$. Let f_j be iso-elastic. Then the firms choices y_j^*, l_j^*, r_j^* and $\mathbf{e}_j^*$ are all increasing in p_j and decreasing in p_w, t_{je}. For $j \in \{b, g\}$ they are decreasing in p_c.

Implications:

- Reducing emissions relative to a status quo necessarily goes together with lower output, employment and abatement.
- Abatement can mitigate but not offset this effect. Complementarity of labor demand and abatement effort:
  - Treating r as a parameter, l^* increases in r.
  - Treating l as a parameter, r^* increases in l.

Competitive equilibrium given policy I

Policy $\mathcal{T}$ consists of

- Consumption taxes $t_x = (t_c, t_g, t_b)$.
- Emission taxes $t_e = (t_{ce}, t_{ge}, t_{be})$.
- A possibly non-linear labor income tax $T_l : p_w y_l \mapsto T_l(p_w y_l)$.

Equilibrium prices for labour, p_w^*, and consumption goods ensure market clearing.

- For consumers denoted by q_c^*, q_g^*, q_b^*.
- For producers denoted by p_c^*, p_g^*, p_b^*.

Proposition 1 (Existence and uniqueness) Zoom in

Under Assumption 1, there is a unique equilibrium price vector.

- Lemma: Walras's Law holds, can set $p_w = 1$.

Competitive equilibrium given policy II

Proposition 2 (Tax incidence)

Under Assumption 1:

1. $t_c \uparrow \Rightarrow p_c \downarrow$, $q_c \uparrow$.
2. $t_{ce} \uparrow \Rightarrow p_c, q_c \uparrow$.
3. $t_{ge} \uparrow$ or $t_{be} \uparrow \Rightarrow p_g, q_g, p_b, q_b \uparrow$.
4. $t_b \uparrow \Rightarrow p_b \downarrow$, $p_g, q_g, q_b \uparrow$.
5. $t_g \uparrow \Rightarrow p_g \downarrow$, $q_g, p_b, q_g \uparrow$.

Proposition 3 (More socially responsible consumers)

Under Assumption 1, when "β increases", then:

1. $p_g, q_g \uparrow$.
2. $p_b, q_b \downarrow$.

Consequence: the green sector becomes greener, the brown sector browner.

A first best benchmark I

Let there be a given utility profile $U_0 : \theta \mapsto U_0(\theta)$. We say that an allocation is first best if it is physically feasible and **reaches this utility profile with minimal emissions.**

Proposition 4 (First-best benchmark) Zoom in

At a solution to a first-best problem:

i) The marginal costs of abatement are equalized across firms and sectors.

ii) The marginal rates of substitution between any pair of consumption goods are equalized across households.

iii) The marginal rates of substitution between consumption goods and effort costs are equalized across households.

Corollary: With sector specific CO2 prices, differential commodity taxation or non-linear income taxation, a competitive equilibrium allocation is not first best.

But: First best allocations are typically not incentive-compatible. And those that are have distributive implications which may be problematic.

A second-best benchmark I

Assumption AS (Atkinson-Stiglitz, 1976): β is the same for all; s is the same for all. $\Rightarrow$ Individuals differ only in ω.

Second-best problem: Add an incentive compatibility constraint to the first-best problem. For any pair ω, ω', $U_0(\omega) \quad \geq \quad u_0(\omega') - k(y_l(\omega'), \omega)$.

Proposition 5 (Heterogeneity only in productive abilities) Zoom in

Under Assumption AS, at a solution to a second-best problem:

i) The marginal costs of abatement are equalized across firms and across sectors.

ii) The marginal rates of substitution between any pair of consumption goods are equalized across households.

Corollary: With sector specific CO2 prices or differential commodity taxation a competitive equilibrium allocation is not second-best. Non-linear income taxation is no impediment for reaching a second-best outcome.

But: Assumption AS is interesting as a benchmark, not empirically plausible.

Uniform price of Carbon: A good idea? I

The emission target. We assume that there is a national emission target $\bar{\mathcal{E}}$:

$$\mathcal{E}(p^*(\mathcal{T}), t_e) \leq \bar{\mathcal{E}} \, . \tag{2}$$

where

$$\mathcal{E}(p^*(\mathcal{T}), t_e) := \sum_{j \in \{c,g,b\}} \mathbf{E_j}[\mathbf{e}_j^*(p^*(\mathcal{T}), t_{je}, \theta_j)]$$

Uniform price of Carbon: A good idea? II

Measures of social welfare.

$$\mathcal{W} = \mathbf{E}_\theta[g(\theta)\, U(\theta)]\ .$$

Shorthands for later use:

- Social marginal utility of disposable income for type θ:

  $$\mathbf{g}(\tilde{v}(\beta, q_x), \theta) := g(\theta)\ \tilde{v}(\beta, q)\,,$$

  where $\tilde{v}(\beta, q)$ is the marginal utility of disposable income.
- Population average of the social marginal utility of income $\bar{\mathbf{g}}$.
- Average amongst recipients of “capital income” from sector j: $\bar{\mathbf{g}}_{\Pi^j}$.

Uniform price of Carbon: A good idea? III

The thought experiment. Start from an allocation induced by a competitive equilibrium with a uniform price of Carbon. Then consider departures from uniform emission taxes and/ or uniform commodity taxes that respect the emission target.

Formally, let $\tau_1 \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$ and $\tau_2 \in \{t_c, t_g, t_b, t_{ce}, t_{ge}, t_{be}\}$ be two different tax rates. A policy change that respects the emission target needs to satisfy

$$\mathcal{E}_{\tau_1} d\tau_1 \ + \ \mathcal{E}_{\tau_2} d\tau_2 = 0$$

or

$$\frac{d\tau_2}{d\tau_1} \quad = \quad -\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \ .$$

Let $d\tau_1 > 0$ and $d\tau_2 < 0$. The welfare-implications of such a policy change are positive if

$$\mathcal{W}_{\tau_1} \ - \ \mathcal{W}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) > 0 \ .$$

Welfare implications of policy changes I

Proposition 6 (Sufficient statistics formula)

Let $\tau \in \{t_c, t_b, t_g, t_{ce}, t_{be}, t_{ge}\}$.

$$
\begin{aligned}
\mathcal{W}_\tau \quad = \quad & -\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau}\ \mathsf{Cov}(\mathbf{g}(\tilde{v}(\cdot), \theta), x_j^*(\cdot)) \\
& + \bar{\mathbf{g}}\ \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T})) \frac{dX_j^*(\cdot)}{d\tau} \\
& + \frac{dp_c^*(\mathcal{T})}{d\tau_j} \Big((\bar{\mathbf{g}}_{\Pi_c} - \bar{\mathbf{g}}) Y_c^*(\cdot) - \sum_j (\bar{\mathbf{g}}_{\Pi_j} - \bar{\mathbf{g}}) \mathbf{E}_j[\gamma_j\ r_j^*(\cdot)] \Big) \\
& + \frac{dp_g^*(\mathcal{T})}{d\tau_j}\ (\bar{\mathbf{g}}_{\Pi_g} - \bar{\mathbf{g}}) X_g^*(\cdot) \quad + \frac{dp_b^*(\mathcal{T})}{d\tau_j}\ (\bar{\mathbf{g}}_{\Pi_b} - \bar{\mathbf{g}}) X_b^*(\cdot) \\
& + \bar{\mathbf{g}}\ \mathbf{E}_\theta [T_l'(y_l^*(\cdot)) y_{l\tau}^*(\cdot)] \\
& + \bar{\mathbf{g}}\ \sum_j t_{je} \frac{d\mathcal{E}_j^*(\cdot)}{d\tau} \\
& + \sum_j \mathbf{I}(\tau = \tau_{je})(\bar{\mathbf{g}} - \bar{\mathbf{g}}_{\Pi_j}) \mathcal{E}_j^*(\cdot)
\end{aligned}
$$

Welfare implications of policy changes II

Equity: Distributive effects across different households

$$
\begin{aligned}
\mathcal{W}_\tau \quad = \quad & -\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau} \, \mathsf{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta), x_j^*(\cdot)) \\
& + \bar{\mathbf{g}} \, \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T})) \frac{dX_j^*(\cdot)}{d\tau} \\
& + \frac{dp_c^*(\mathcal{T})}{d\tau_j} \Big((\bar{\mathbf{g}}_{\Pi_c} - \bar{\mathbf{g}}) Y_c^*(\cdot) - \sum_j (\bar{\mathbf{g}}_{\Pi_j} - \bar{\mathbf{g}}) \mathbf{E}_j [\gamma_j \; r_j^*(\cdot)] \Big) \\
& + \frac{dp_g^*(\mathcal{T})}{d\tau_j} \, (\bar{\mathbf{g}}_{\Pi_g} - \bar{\mathbf{g}}) X_g^*(\cdot) \quad + \frac{dp_b^*(\mathcal{T})}{d\tau_j} \, (\bar{\mathbf{g}}_{\Pi_b} - \bar{\mathbf{g}}) X_b^*(\cdot) \\
& + \bar{\mathbf{g}} \, \mathbf{E}_\theta [T_l'(y^*(\cdot)) y_{l\tau}^*(\cdot)] \\
& + \bar{\mathbf{g}} \, \sum_j t_{je} \frac{d\mathcal{E}_j^*(\cdot)}{d\tau} \\
& + \sum_j \mathbf{I}(\tau = \tau_{je}) (\bar{\mathbf{g}} - \bar{\mathbf{g}}_{\Pi_j}) \mathcal{E}_j^*(\cdot)
\end{aligned}
$$

Welfare implications of policy changes III

Efficiency: Change of equilibrium quantities – Behavioral responses

$$\mathcal{W}_\tau = -\sum_j \frac{dq_j^*(\mathcal{T})}{d\tau} \text{Cov}(\mathbf{g}(\tilde{v}(\cdot),\theta), x_j^*(\cdot))$$

$$+\bar{\mathbf{g}} \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T})) \frac{dX_j^*(\cdot)}{d\tau}$$

$$+\frac{dp_c^*(\mathcal{T})}{d\tau_j}\Big((\bar{\mathbf{g}}_{\Pi_c} - \bar{\mathbf{g}})Y_c^*(\cdot) - \sum_j (\bar{\mathbf{g}}_{\Pi_j} - \bar{\mathbf{g}})\mathbf{E}_j[\gamma_j\, r_j^*(\cdot)]\Big)$$

$$+\frac{dp_g^*(\mathcal{T})}{d\tau_j}(\bar{\mathbf{g}}_{\Pi_g} - \bar{\mathbf{g}})X_g^*(\cdot) \quad + \frac{dp_b^*(\mathcal{T})}{d\tau_j}(\bar{\mathbf{g}}_{\Pi_b} - \bar{\mathbf{g}})X_b^*(\cdot)$$

$$+\bar{\mathbf{g}}\, \mathbf{E}_\theta[T_l'(y^*(\cdot))y_{l\tau}^*(\cdot)]$$

$$+\bar{\mathbf{g}} \sum_j t_{je} \frac{d\mathcal{E}_j^*(\cdot)}{d\tau}$$

$$+\sum_j \mathbf{I}(\tau = \tau_{je})(\bar{\mathbf{g}} - \bar{\mathbf{g}}_{\Pi_j})\mathcal{E}_j^*(\cdot)$$

Departing from a uniform price of Carbon I

- Uniform commodity taxation in the status quo:

$$\bar{\mathbf{g}} \sum_j (q_j^*(\mathcal{T}) - p_j^*(\mathcal{T})) \frac{dX_j^*(\cdot)}{d\tau} \quad = \quad 0 \, .$$

- Uniform CO2 price in the status quo: Let $t_{je} = \bar{t}_e$, for all j.
- **Distributive indifference:** For all θ, $\mathbf{g}(\tilde{v}(\cdot), \theta) = \bar{\mathbf{g}}$.
- **Proportional fiscal externalites:** There is a number η, so that, for all $\tau \in \{t_c, t_b, t_g, t_{ce}, t_{be}, t_{ge}\}$,

$$\frac{\mathbf{E}_\theta[T_l'(y_l^*(\cdot, \theta)) y_{l\tau}^*(\cdot, \theta)]}{\mathcal{E}_\tau} \quad = \quad \eta \, ,$$

Under these assumptions

$$\mathcal{W}_\tau \quad = \quad \bar{\mathbf{g}} \, (\bar{t}_e + \eta) \, \mathcal{E}_\tau \, .$$

Departing from a uniform price of Carbon II

Apply test for the desirability of a uniform Carbon price: With

$$\mathcal{W}_{\tau} = \bar{\mathbf{g}} \left(\bar{t}_e + \eta \right) \mathcal{E}_{\tau}$$

have

$$\mathcal{W}_{\tau_1} \; - \; \mathcal{W}_{\tau_2} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) = 0 \; .$$

Hence, no welfare-gain from departing from the market-based approach.

Proposition 7 (Suff. conditions for a uniform Carbon price)

With distributive indifference and proportional fiscal externalities there are no welfare gains from deviating from a uniform price of Carbon.

Departing from a uniform price of Carbon III

1. Is the assumption of distributive indifference normatively appealing?

   Suppose that s or β and ω are positively correlated. Then it is incompatible with weights that are higher for people with lower disposable income.

2. Is the assumption of proportional fiscal externalities empirically plausible ?

   Broadly, what has a small impact on earnings incentives also has a small impact on overall emissions.

   Counterexamples are conceivable: E.g. limited GE effects of taxes on prices, zero emissions for the green good $\Rightarrow$ tax the green good more than the brown good.

Can a uniform Carbon price be desirable under alternative assumptions?

- Write $\mathcal{W}_\tau \quad = \quad \mathcal{W}_\tau^{net} \, + \, \bar{\mathbf{g}} \, \sum_j t_{je} \frac{d\mathcal{E}_j^*(\cdot)}{d\tau}$
- Then, starting from a uniform price, the welfare impact of raising some tax rate τ_1 and lowering some tax rate τ_2 is given by

$$\mathcal{W}_{\tau_1}^{net} - \mathcal{W}_{\tau_2}^{net} \left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}} \right) .$$

  It is possible that this expression is (close to) zero even if $\mathcal{W}_{\tau_1}^{net} \neq 0$ and $\mathcal{W}_{\tau_2}^{net} \neq 0$.
- An empirical application of the sufficient statistics approach would tell.
- Question of welfare weights and elasticities – as opposed to principles of climate policy design.

Equity considerations I

Approach:

- Consider welfare weights that are monotonic in disposable income.
- Again, we consider the equ. that results with $t_{ce} = t_{be} = t_{ge} =: \bar{t}_e$, and $t_c = t_b = t_g = 0$. Again, check whether, for any pair τ_1, τ_2,

$$\mathcal{W}^{net}_{\tau_1} - \mathcal{W}^{net}_{\tau_2}\left(\frac{\mathcal{E}_{\tau_1}}{\mathcal{E}_{\tau_2}}\right) \quad = \quad 0\,, \tag{3}$$

- Special case of the more general model developed in Section 2. Firms operate with constant returns to scale technologies and fixed emission intensities.

  $\Rightarrow$ Producer prices are fixed $p_j = 1 + t_{je}\, e_{j0}$.

  $\Rightarrow$ Tax increases fully passed to consumers, $q_j = (1 + t_j)(1 + t_{je}\, e_{j0})$.

Equity considerations II

Proposition 8 (Rich vs Poor rather then Brown vs Green I)

Consider the competitive equilibrium allocation that results with a uniform price of Carbon and suppose that fiscal externalities are proportional. Consider two goods $j, k \in \{c, g, b\}$ so that

$$\text{Cov}(\mathbf{g}(\cdot), x_k^*(\cdot)) \quad < \quad 0 \quad < \quad \text{Cov}(\mathbf{g}(\cdot), x_j^*(\cdot)) \ .$$

Welfare goes up when t_{ke} or t_k are increased and t_{je} or t_j are decreased.

Equity considerations III

Proposition 9 (Rich vs Poor rather then Brown vs Green II)

Suppose Assumption 1 holds

1. Let $\mathcal{W}_\tau^{net} = 0$ for $\tau \in \{t_c, t_{ce}\}$. Let Cov$(\mathbf{g}(\cdot), x_j^*(\cdot)) <$ Cov$(\mathbf{g}(\cdot), x_c^*(\cdot))$. Then $\mathcal{W}_\tau^{net} > 0$ for $\tau \in \{t_j, t_{je}\}$ and $\mathcal{W}_\tau^{net} < 0$ for $\tau \in \{t_{-j}, t_{-je}\}$.
2. Suppose that for all individuals β takes the same value, henceforth denoted by $\bar{\beta}$. Then, for any pair $\tau_1, \tau_2 \in \{t_{ce}, t_{ge}, t_{be}, t_c, t_g, t_b\}$

$$\text{sgn}\, \mathcal{W}_{\tau_1}^{net} \quad = \quad \text{sgn}\, \mathcal{W}_{\tau_2}^{net} \,.$$

Concluding remarks I

Summary:

- Climate policy is confronted with an equity-efficiency trade-off.
- A uniform price on Carbon is efficient in the sense that it allows to reach national emission targets at minimal costs.
- Deviations to a sector-specific climate policy can be justified by distributive concerns.
- In the presence of non-linear income taxes, a second-best logic may imply that deviations from a uniform price of Carbon can be justified by efficiency considerations.

Concluding remarks II

Outside the model:

- A uniform Carbon price has advantages of simplicity and accountability. Those are not captured by the welfare analysis that is presented in this paper.
- As suggested by the welfare analysis in this paper, the distributive implications of such an approach may be perceived as unfair.
- Possibly this is an explanation for the lack of political support and the protests that are spurred by plans for more ambitious climate policies.
- Reaching emission targets in a politically feasible way may therefore require a sector-specific approach.

Competitive equilibrium given policy I

Definition: Given a policy $\mathcal{T}$, a price system $p = (p_w, p_c, p_g, p_b)$ is an equilibrium price system if the following equilibrium conditions are met:

- The labor market clears,

$$L(p, t_e) = Y_l(\Pi^E, \mathcal{R}^E, q_x, p_w, T_l) \ ,$$

- The goods markets clear

$$X_c(\Pi^E, \mathcal{R}^E, q_x, p_w, T_l) + R(p, t_e) = Y_c(p_c, p_w, t_{ce}) \ ,$$

$$X_g(\Pi^E, \mathcal{R}^E, q_x, p_w, T_l) = Y_g(p_g, p_c, p_w, t_{ge}) \ ,$$

  and

$$X_b(\Pi^E, \mathcal{R}^E, q_x, p_w, T_l) = Y_b(p_b, p_c, p_w, t_{be}) \ ,$$

- Expectations are correct

$$\mathcal{R}^E = \mathcal{R}(p, T) \quad \text{and} \quad \Pi^E = \Pi(p, t_e) \, ,$$

  where $\Pi(p, T)$ is the vector of profits in the different sectors of the economy,

for $q_x = (q_c, q_g, q_b) = \Big((1 + t_c)p_c, (1 + t_g)p_g, (1 + t_b)p_b\Big)$.

Equilibrium existence I

Sketch of proof.

- Lemma: If the goods markets clear and expectations are correct, then the labor market clears.

  $\Rightarrow$ Set $p_w = 1$ and work with the three goods market clearing conditions.

- With ν small, labor market outcomes, disposable incomes and earnings do not depend much on the prices of the green and the brown good.
- Under these assumptions, the excess demand functions constructed in the proof of the Proposition have the property that they are monotonically decreasing in the "own" price, while still depending, in a parametric way, on the prices of the other goods.
- How to find the equilibrium price vector? Sequential market clearing.

Equilibrium existence II

Sequential market clearing

1. Fix p_b and p_c at arbitrary levels and show that there is unique price p_g that clears the market for the green good.
2. As we vary p_b, this partial equilibrium value of p_g adjusts. So, we can vary both p_b and p_g so that the green market remains in partial equilibrium.
3. We note that any such variation that involves a higher/ lower level of p_b lowers/ increases excess demand in the market for the brown good. Thus, we can bring the market for the brown good into partial equilibrium, while maintaining the partial equilibrium in the market for the green good.
4. All this holds for arbitrary values of p_c. As a final step we bring p_c to the level that clears the market for the unspecific consumption good while adjusting p_g and p_b so that both the market for the green good and the market for the brown good both remain in partial equilibrium.

Ultimately we have found a general equilibrium price vector in this way. Zoom out

A first best benchmark I

Let there be a given utility profile $U_0 : \theta \mapsto U_0(\theta)$. We say that an allocation is first best if it is physically feasible and reaches this utility profile with minimal emissions.

Formally, a first-best allocation solves the following:

Choose

- Household labor supply $y_l : \theta \mapsto y_l(\theta)$ and consumption $x_c : \theta \mapsto x_c(\theta)$, $x_b : \theta \mapsto x_b(\theta)$ and $x_g : \theta \mapsto x_g(\theta)$.
- For every sector j and every type of firm $\theta_j = (\alpha_j, \gamma_j)$ labor inputs and $R\&D$ effort: $l_j : \theta_j \mapsto l_j(\theta_j)$ and $r_j : \theta_j \mapsto r_j(\theta_j)$.

to minimize

$$\sum_{j \in \{c,b,g\}} \mathcal{E}_j \quad = \quad \sum_{j \in \{c,b,g\}} \mathbf{E}_j \left[\Big(e_{0j} - a_j(r_j(\theta_j)) \Big) \alpha_j f_j(l_j(\theta_j)) \right] \ .$$

A first best benchmark II

Constraints:

- The chosen allocation needs to reach utility profile U_0.

$$u(x_c(\theta), \chi(\beta x_g(\theta), x_b(\theta))) - k(y(\theta), \omega) = U_0(\theta) \ . \tag{4}$$

- The labour used up in the production process is bounded from above by the amount that households make available,

$$\sum_{j \in \{c,b,g\}} \mathbf{E}_j \left[l_j(\theta_j) \right] \quad \leq \quad \mathbf{E}_\theta \left[y_l(\theta) \right] \ . \tag{5}$$

- Aggregate consumption is bounded by the production sector's (net) output

$$\mathbf{E}_\omega[x_c(\omega)] \quad \leq \quad \mathbf{E}_c[\alpha_c f_c(l_c(\theta_c))] - \sum_{j \in \{c,b,g\}} \mathbf{E}_j[\gamma_j r_j(\theta_j)] \ , \tag{6}$$

$$\mathbf{E}_\omega[x_g(\omega)] \quad \leq \quad \mathbf{E}_g[\alpha_g f_g(l_g(\theta_g))] \ , \tag{7}$$

and

$$\mathbf{E}_\omega[x_b(\omega)] \quad \leq \quad \mathbf{E}_b[\alpha_b f_b(l_b(\theta_b))] \ . \tag{8}$$

A first best benchmark III

Proposition 4 Zoom out

At a solution to a first-best problem:

i) The marginal costs of emission avoidance are equalized: For any $j, k \in \{c, g, b\}$, and any pair $\theta_j = (\alpha_j, \gamma_j)$ and $\theta_k = (\alpha_k, \gamma_k)$,

$$\frac{\gamma_j}{a_j'(r_j(\theta_j))\alpha_j f_j(l_j(\theta_j))} = \frac{\gamma_k}{a_k'(r_k(\theta_k))\alpha_k f_k(l_k(\theta_k))} \,. \tag{9}$$

ii) The marginal rates of substitution between any pair of consumption goods are equalized across households.

iii) The marginal rates of substitution between consumption goods and effort costs are equalized across households.

Corollary: With sector specific CO2 prices, differential commodity taxation or non-linear income taxation, a competitive equilibrium allocation is not first best.

But: First best allocations are typically not incentive-compatible. And those that are have distributive implications which may be problematic.

A second-best benchmark I

Assumption AS (Atkinson-Stiglitz, 1976)

- All individuals have the same preferences over consumption goods, i.e. β is the same for all.
- All individuals have identical claims on the profits generated in the economy, i.e. s is the same for all.
- Individuals differ only in their productive abilities ω.

Second-best problem: Add an incentive compatibility constraint to the first-best problem. For any pair ω, ω',

$$U_0(\omega) \quad \geq \quad u_0(\omega') - k(y_l(\omega'), \omega) \; .$$

where $u_0(\omega') := u(x_c(\omega'), \chi(\beta x_g(\omega'), x_b(\omega')))$.

A second-best benchmark II

Proposition 5 Zoom out

Under Assumption AS, at a solution to a second-best problem:

i) The marginal costs of emission avoidance are equalized: For any $j, k \in \{c, g, b\}$, and any pair $\theta_j = (\alpha_j, \gamma_j)$ and $\theta_k = (\alpha_k, \gamma_k)$,

$$\frac{\gamma_j}{a_j'(r_j(\theta_j))\alpha_j f_j(l_j(\theta_j))} = \frac{\gamma_k}{a_k'(r_k(\theta_k))\alpha_k f_k(l_k(\theta_k))} \,. \tag{10}$$

ii) The marginal rates of substitution between any pair of consumption goods are equalized across households.

Corollary: With sector specific CO2 prices or differential commodity taxation a competitive equilibrium allocation is not second-best. Non-linear income taxation is no impediment for reaching a second-best outcome.

But: Assumption AS is interesting as a benchmark, not empirically plausible.